SS – 348

III Semester B.A./B.Sc. Examination, Nov./Dec. 2018
(NS) (2012-13 and Onwards) (Repeaters – Prior to 2015-16)
MATHEMATICS
Mathematics – III

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Define a normal subgroup.

2) Prove that every subgroup of an abelian group is normal.

3) If $f : (z, +) \to (z, +)$ is defined by $f(x) = -x$, $\forall x \in z$ then show that f is homomorphism.

4) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ then find $f \circ g$.

5) Draw the graph of the system of inequalities $x \geq 3$ and $x \leq 6$.

6) Define basic feasible solution and optimal solution.

7) Show that $X = \{x : |x| \leq 3\}$ is a convex set.

8) Show that the sequence $\left\{\frac{1}{n}\right\}$ is bounded.

9) Show that the sequence whose n^{th} term is $\frac{2n+5}{3n+4}$ is monotonically decreasing.

10) Test the convergence of the sequence whose n^{th} term is $1 + \cos n\pi$.

11) Find the limit of a sequence whose n^{th} term is $\frac{n^2+3n+7}{(2n-3)(n+5)}$.

12) Discuss the convergence of the series $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots$.

13) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^4+1}$.

P.T.O.

14) State Leibnitz's test for alternating series.

15) State Cauchy's root test for series of positive terms.

16) Sum to infinity the series $\frac{x}{\lfloor 1} + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \ldots$ to ∞.

17) If $f(x) = \begin{cases} 1+x & \text{for} \quad x<2 \\ K & \text{for} \quad x=2 \\ 5-x & \text{for} \quad x>2 \end{cases}$

is continuous at x = 2, find K.

18) State Rolle's theorem.

19) Expand log (1 + x) up to the term containing x^2 by Maclaurins theorem.

20) Evaluate $\lim_{x \to 0} \frac{e^x + \sin x - 1}{\log(1+x)}$.

II. Answer **any two** questions : **(2×5=10)**

1) Let H be a subgroup and K be a normal subgroup of a group G. Show that $H \cap K$ is normal in H.

2) Prove that Homomorphic image of an abelian group is abelian.

3) Let $f : G \to G'$ be a homomorphism of a group with Kernal K, then prove that f is one-one if and only if K = {e} where 'e' is the identity element of G.

4) Show that every infinite cyclic group is isomorphic to additive group of integers.

III. Answer **any three** questions : **(3×5=15)**

1) Prove that the set of all feasible solution of LPP is a convex set.

2) Find all basic feasible solution of the system of equation x + 2y + z = 4 and 2x + y + 5z = 5.

3) Using simplex method maximize P = 5x + y + 4z subject to the constraints $y + z \leq 3$, $x + y + z \leq 5$ $x + z = 8$, $x \geq 0$, $y \geq 0$, $z \geq 0$.

4) Using Vogel's Method, find the minimum cost of transportation from

		D1	D2	D3	D4	Supply
Origin	01	11	13	17	14	250
	02	16	18	14	10	300
	03	21	24	13	10	400
	Demand	200	225	275	250	950

IV. Answer **any two** questions : **(2×5=10)**

1) Find the limit of a sequence

i) $\{(-1)^n\}$

ii) $\left\{\frac{2n+3}{5n+6}\right\}$.

2) Prove that a monotonically increasing sequence bounded above is convergent.

3) Discuss the nature of the sequence $\left\{n^{1/n}\right\}$.

V. Answer **any four** questions : **(4×5=20)**

1) Discuss the convergence of the series $\frac{x}{2.3}+\frac{x^2}{3.4}+\frac{x^3}{4.5}+\ldots \text{ to } \infty \, (x>0)$.

2) State and prove D'Alembert's ratio test.

3) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$.

4) Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n-1}\left[\sqrt{n+1}-\sqrt{n}\right]$.

5) Sum the series $1+\frac{3}{3}+\frac{3.5}{3.6}+\frac{3.5.7}{3.6.9}+\ldots$.

6) Sum the series $\sum_{n=1}^{\infty} \frac{(3n^2-n+1)}{\underline{|n}} x^n$.

VI. Answer **any three** questions : **(3×5=15)**

1) Examine the differentiability of the function $f(x) = \begin{cases} 5x - 4 & \text{if } 0 < x \le 1 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \end{cases}$ at x = 1.

2) State and prove Lagrange's mean value theorem.

3) Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in [2, 4].

4) Expand log (1 – x) upto the term containing x^4 by Maclaurins theorem.

5) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.